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On the physical Hilbert space of QCD_2 in the decoupled formulation

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Abstract

We consider the QCD_2 partition function in the non-local, decoupled formulation and systematically establish which subset of the nilpotent Noether charges is required to vanish on the physical states. The implications for the Hilbert space structure are also examined.

1 Introduction

The formulation of two-dimensional quantum chromodynamics (QCD_2) of massless fermions in terms of decoupled fermions, ghosts and positive and negative level Wess-Zumino fields [1, 2, 3] has provided interesting insight into some non-perturbative properties of this theory. Two representations of the corresponding decoupled partition function, referred to as "local" and "non-local" representations [2, 3], have been considered. In the "local" formulation, the original restriction of the "observables" to the gauge invariant subspace of the Hilbert space is replaced in the light-cone gauge $A_+ \equiv A_0 + A_1 = 0$ by the requirement that "observables" commute with two BRST charges [4, 5].

When passing to the "non-local" formulation, one expects to pick up one additional BRST condition associated with the change of variable involved in the transition. De facto one finds, however, more than three nilpotent charges, which are moreover non-commuting. This raises the question as to which of these charges are required to annihilate the physical states. This question has recently been addressed in the context of quantum mechanical toy models and the <u>local</u> decoupled formulation of QCD_2, in ref.[5], where criteria have also been given for establishing which BRST conditions should actually be imposed.

The primary aim of the present paper is to examine this question for the case of QCD_2 in the non-local decoupled formulation. As we show in section 2, not all of the nilpotent charges obtained in ref.[4] are required to vanish on the physical Hilbert space $\mathcal{H}_{\text{phys}}$. In section 3 we solve the corresponding cohomology problem in the ghost number zero sector by showing that the BRST conditions which are actually to be imposed are implemented by the gauge invariant observables of the theory.

In section 4, we then discuss the conformal "sector" of the factorized non-local partition function. In ref.[6] the QCD_2 ground state was taken to lie in this sector. It was thereby concluded that in the case of one flavor and gauge group $SU(2)$ the QCD_2 ground state is two-fold degenerate. We generalize this statement to the case of $SU(N)$-color.

Section 5 summarizes our results.

2 BRST constraints

We reconsider here the BRST analysis of ref.[4]. We discuss separately the "local" [1, 2] and "non-local" formulations [2, 3].

2.1 Local formulation

The QCD_2 partition function is given by

$$Z = \int \mathcal{D}A_\mu \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp[-i\int \frac{1}{4} trF_{\mu\nu}F^{\mu\nu}] \exp[i\int \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi], \quad (2.1)$$

where $F_{\mu\nu}$ is the chromoelectric field strength tensor. Going to the light-cone gauge $A_+ = 0$, parametrizing A_- as

$$eA_- = Vi\partial_- V^{-1}, \quad (2.2)$$

and performing a chiral rotation, $\psi_2 = V\psi_2^{(0)}$, one arrives at the decoupled partition function (for details see for instance ref.[4])

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_V, \quad (2.3)$$

where $Z_F^{(0)}$, $Z_{gh}^{(0)}$ are the partition functions of massless free fermions and ghosts, respectively,

$$\begin{aligned} Z_F^{(0)} &= \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp[i\int \bar{\psi}^{(0)} i\partial\!\!\!/ \psi^{(0)}], \\ Z_{gh}^{(0)} &= \int \mathcal{D}(\text{ghosts}) \exp\left\{i\int [b_+^{(0)} i\partial_- c_+^{(0)} + b_-^{(0)} i\partial_+ c_-^{(0)}]\right\}, \end{aligned} \quad (2.4)$$

and

$$Z_V = \int \mathcal{D}V \exp\left\{-i(1+c_V)\Gamma[V] + \frac{i}{8e^2}\int d^2x tr[\partial_+(Vi\partial_- V^{-1})]^2\right\}. \quad (2.5)$$

Here $\Gamma[V]$ is the usual Wess-Zumino-Witten (WZW) functional [7], and c_V is the Casimir of the gauge group.

As shown in ref.[4] this decoupled partition function exhibits two BRST symmetries implying the existence of two nilpotent Noether charges. The corresponding BRST currents are [4]

$$\mathcal{J}_\pm = trc_\pm^{(0)}\left[\Omega_\pm - \frac{1}{2}\left\{b_\pm^{(0)}, c_\pm^{(0)}\right\}\right], \quad (2.6)$$

where

$$\begin{aligned} \Omega_- &= -\frac{1}{4e^2}\mathcal{D}_-(V)\partial_+(Vi\partial_- V^{-1}) - (1+c_V)J_-(V) + j_-, \\ \Omega_+ &= -\frac{1}{4e^2}\mathcal{D}_+(V)\partial_-(V^{-1}i\partial_+ V) - (1+c_V)J_+(V) + j_+. \end{aligned} \quad (2.7)$$

Here $\mathcal{D}_\pm(V)$ are the covariant derivatives

$$\begin{aligned} \mathcal{D}_+(V) &= \partial_+ + [V^{-1}\partial_+ V, \ ], \\ \mathcal{D}_-(V) &= \partial_- + [V\partial_- V^{-1}, \ ] \end{aligned} \quad (2.8)$$

and use has been made of the identity

$$V^{-1}\left[\partial_+^2(V\partial_-V^{-1})\right]V = \mathcal{D}_+(V)\partial_-(V^{-1}\partial_+V), \tag{2.9}$$

in order to write the BRST currents of ref.[4] in symmetrical form. $J_\pm(V)$ and $j_\pm$ are the currents

$$\begin{aligned}
J_+(V) &= \frac{1}{4\pi}V^{-1}i\partial_+V,\\
J_-(V) &= \frac{1}{4\pi}Vi\partial_-V^{-1},\\
j_- &= \psi_1^{(0)}\psi_1^{(0)\dagger} + \left\{b_-^{(0)}, c_-^{(0)}\right\},\\
j_+ &= \psi_2^{(0)}\psi_2^{(0)\dagger} + \left\{b_+^{(0)}, c_+^{(0)}\right\}.
\end{aligned} \tag{2.10}$$

The gauge invariance of the observables in the original formulation (2.1) is replaced in the decoupled picture by the requirement that the physical operators commute with the BRST charges $Q_\pm$ associated with $\mathcal{J}_\pm$ (see [5] for proof). In the ghost number zero sector this implies that $\Omega_\pm$ defined by (2.7) are constrained to vanish on the physical subspace:

$$\Omega_\pm \approx 0. \tag{2.11}$$

As shown in ref.[8], this property can independently be established by appropriately gauging the action in the decoupled partition function.

2.2 Non-local formulation

The main objective of this paper is to trace the fate of the BRST conditions of the local formulation, when going over to the so-called [2] "non-local" formulation, and to establish from first principles further BRST conditions that may have to be imposed in order to ensure equivalence of this formulation with the local one.

In order to make the discussion self-contained, we repeat here the essential steps leading to the non-local formulation [2, 3, 4]. We rewrite the partition function Z_V given in (2.5) by making use of the identity

$$\begin{aligned}
&\exp\left\{\frac{i}{4e^2}\int tr\frac{1}{2}[\partial_+(Vi\partial_-V^{-1})]^2\right\}\\
&\quad= \int \mathcal{D}E\exp\left\{-i\int tr\left[\frac{1}{2}E^2 + \frac{E}{2e}\partial_+(Vi\partial_-V^{-1})\right]\right\}.
\end{aligned} \tag{2.12}$$

Making the change of variable

$$\partial_+E = \lambda\beta^{-1}i\partial_+\beta\ ,\ \lambda = \left(\frac{1+c_V}{2\pi}\right)e, \tag{2.13}$$

we have for the corresponding change in the integration measure

$$\mathcal{D}E = e^{-ic_V\Gamma[\beta]}\mathcal{D}\beta. \tag{2.14}$$

Making use of the Polyakov-Wiegmann identity [9]

$$\Gamma[gh] = \Gamma[g] + \Gamma[h] + \frac{1}{4\pi}\int d^2x tr\left(g^{-1}\partial_+ g h\partial_- h^{-1}\right), \tag{2.15}$$

and defining the new variable

$$\tilde{V} = \beta V, \tag{2.16}$$

one then arrives at a decoupled non-local form of the partition function [2, 4]:

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_{\tilde{V}} Z_\beta, \tag{2.17}$$

where

$$Z_{\tilde{V}} = \int \mathcal{D}\tilde{V} e^{-i(1+c_V)\Gamma[\tilde{V}]}, \tag{2.18}$$

and

$$Z_\beta = \int \mathcal{D}\beta e^{i\Gamma[\beta]+i\lambda^2\int\frac{1}{2}tr[\partial_+^{-1}(\beta^{-1}\partial_+\beta)]^2}. \tag{2.19}$$

We now investigate the BRST conditions to be imposed on the physical states in this formulation.

a) BRST condition associated with the change of variable $E \rightarrow \beta$

We begin by showing that the change of variable (2.13) leads to a BRST condition on the physical states. We follow the procedure outlined in ref.[10].

In order to implement the change of variable (2.13), we introduce in (2.12) the identity

$$\lambda\int \mathcal{D}\beta det\mathcal{D}_+(\beta)\delta\left[\partial_+ E - \lambda\beta^{-1}i\partial_+\beta\right] = 1, \tag{2.20}$$

in order to rewrite (2.5) in the form

$$Z_V = \int\mathcal{D}V\int\mathcal{D}E\mathcal{D}\beta\int\mathcal{D}\rho\int\mathcal{D}(ghosts)e^{iS[E,V]}e^{i\Delta S[E,\beta,\rho,ghosts]}, \tag{2.21}$$

where

$$S[E,V] = -(1+c_V)\Gamma[V] - \int d^2x[\frac{1}{2}E^2 + \frac{E}{2e}\partial_+(Vi\partial_- V^{-1})] \tag{2.22}$$

and

$$\Delta S[E,\beta,\rho,ghosts] = \int d^2x\{\rho(\partial_+ E - \lambda\beta^{-1}i\partial_+\beta) + \hat{b}_- i\mathcal{D}_+(\beta)\hat{c}_-\}. \tag{2.23}$$

Here we have made use of the Fourier representation of the δ functional in (2.20), and the representation of the adjoint determinant $det\mathcal{D}_+(\beta)$ in terms of ghosts:

$$det\mathcal{D}_+(\beta) = \int \mathcal{D}\hat{b}\mathcal{D}\hat{c}e^{i\int \hat{b}i\mathcal{D}_+(\beta)\hat{c}}. \tag{2.24}$$

The effective action is seen to be invariant under the transformation

$$\begin{aligned}
&\delta V = 0, \quad \delta E = 0, \quad \delta\rho = 0, \\
&\delta\hat{b}_- = \lambda\rho, \quad \delta\hat{c}_- = -\frac{1}{2}\{\hat{c}_-, \hat{c}_-\}, \\
&\beta^{-1}\delta\beta = \hat{c}_-.
\end{aligned} \tag{2.25}$$

One readily checks that these transformations are off-shell nilpotent. We further observe that ΔS in (2.23) can be written as

$$\Delta S = \frac{1}{\lambda}\delta[\hat{b}_-(\partial_+ E - \lambda\beta^{-1}i\partial_+\beta)]. \tag{2.26}$$

Hence ΔS is BRST exact. From here we infer that the actions $S[E,V]$ and $S[E,V]+\Delta S$ are equivalent on the functionals which are invariant under the nilpotent transformations (2.25). Hence physical states must be invariant under the transformations (2.25).

In order to obtain the transformation laws (2.25) in terms of the variables of the non-local formulation, we make use of the equations of motion for ρ and E. The transformations (2.25) then reduce to

$$\begin{aligned}
&\delta V = 0, \\
&\delta\hat{b}_- = -\lambda^2\partial_+^{-2}(\beta^{-1}i\partial_+\beta) - \frac{\lambda}{2e}(Vi\partial_- V^{-1}), \\
&\delta\hat{c}_- = -\frac{1}{2}\{\hat{c}_-, \hat{c}_-\}, \\
&\beta^{-1}\delta\beta = \hat{c}_-.
\end{aligned} \tag{2.27}$$

We next decouple the ghosts by performing the change of variable

$$\begin{aligned}
&\hat{b}_- \rightarrow \beta\hat{b}_-\beta^{-1} =: \hat{b}_-^{(0)}, \\
&\hat{c}_- \rightarrow \beta\hat{c}_-\beta^{-1} =: \hat{c}^{(0)}.
\end{aligned} \tag{2.28}$$

In terms of $\tilde{V} = \beta V$ and the decoupled ghosts, the transformation laws then read

$$\begin{aligned}
\delta\tilde{V}\tilde{V}^{-1} &= \hat{c}_-^{(0)}, \\
\delta\beta\beta^{-1} &= \hat{c}_-^{(0)}, \\
\delta\hat{c}_-^{(0)} &= \frac{1}{2}\{\hat{c}_-^{(0)}, \hat{c}_-^{(0)}\},
\end{aligned}$$

$$\begin{aligned}\delta\hat{b}_-^{(0)} &= -\lambda^2\beta\partial_+^{-2}(\beta^{-1}i\partial_+\beta)\beta^{-1} \\ &\quad -\left(\frac{1+c_V}{4\pi}\right)\left[(\tilde{V}i\partial_-\tilde{V}^{-1})-\beta i\partial_-\beta^{-1}\right] \\ &\quad +\{b_-^{(0)},c_-^{(0)}\}+(\delta\hat{b}_-^{(0)})_{anom},\end{aligned} \tag{2.29}$$

where the anomalous term

$$(\delta\hat{b}_-^{(0)})_{anom} = -\frac{c_V}{4\pi}\beta i\partial_-\beta^{-1}, \tag{2.30}$$

needs to be added to the semiclassical result in order to make the transformation an invariance of the quantum action. As has been shown in ref.[4], the transformation laws (2.29) lead to the (right-moving) BRST current

$$\hat{J}_- = tr\hat{c}_-^{(0)}\left(\hat{\Omega}_- - \frac{1}{2}\{\hat{b}_-^{(0)},\hat{c}_-^{(0)}\}\right), \tag{2.31}$$

with

$$\begin{aligned}\hat{\Omega}_- &= -\lambda^2\beta(\partial_+^{-2}(\beta^{-1}i\partial_+\beta))\beta^{-1}+J_-(\beta) \\ &\quad -(1+c_V)J_-(\tilde{V})+\left\{\hat{b}_-^{(0)},\hat{c}_-^{(0)}\right\}.\end{aligned} \tag{2.32}$$

Our deductive procedure shows that the corresponding nilpotent charge $\hat{Q}_-$ must annihilate the physical states:

$$\hat{Q}_- = 0 \quad \text{on} \quad \mathcal{H}_{phys}. \tag{2.33}$$

b) Fate of the BRST condition $Q_+ \approx 0$

Making use of the identity (2.9), we may rewrite Ω_+ in (2.7) as

$$\Omega_+ = -\frac{1}{4e^2}V^{-1}\left[\partial_+^2(Vi\partial_-V^{-1})\right]V-(1+c_V)J_+(V)+j_+. \tag{2.34}$$

Using the equation of motion for E following from (2.12)

$$E = -\frac{1}{2e}\partial_+(Vi\partial_-V^{-1}), \tag{2.35}$$

Ω_+ takes the form

$$\Omega_+ = \frac{1}{2}V^{-1}(\partial_+E)V-(1+c_V)J_+(V)+j_+. \tag{2.36}$$

Making the change of variable (2.13) and (2.16), we then obtain[1]

$$\Omega_+ = -(1+c_V)J_+(\tilde{V})+j_+, \tag{2.37}$$

[1] For the sake of clarity we continue to use the same notation for the constraints when expressed in terms of the new variables.

where $\tilde{V} = \beta V$. Comparing with eq.(3.13) of ref.[4], we see that this is just $\tilde{\Omega}_+$ of ref.[4]. We conclude that the corresponding nilpotent charge

$$Q_+ = \int dx^1 trc_+^{(0)} \left[-(1+c_V)J_+(\tilde{V}) + j_+ - \frac{1}{2}\left\{ b_+^{(0)}, c_+^{(0)} \right\} \right], \qquad (2.38)$$

must annihilate the physical states, as was also required in ref.[4].

c) Fate of the BRST condition $Q_- \approx 0$

In the case of the BRST charge Q_+, the symmetry transformations in the V-fermion-ghost space giving rise to this conserved charge could be trivially extended to the $E-V$-fermion-ghost space. This is no longer true in the case of Q_-, where the BRST symmetry for E off-shell is maintained only at the expense of the addition of a (commutator) term (which vanishes for E "on shell"). One is thereby led to a fairly complicated expression for Q_- when expressed in terms of the variables $\beta, \tilde{V}$ of the non-local formulation.

A more transparent result is obtained by performing the similarity transformation

$$E' = -V^{-1}EV \qquad (2.39)$$

and making the change of variables

$$\partial_- E' = \lambda \beta' i \partial_- \beta'^{-1}. \qquad (2.40)$$

Going through the same steps as outlined before, one arrives at an alternative representation of the partition function (2.17),

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_{\tilde{V}'} Z_{\beta'}, \qquad (2.41)$$

where

$$Z_{\tilde{V}'} = \int \mathcal{D}\tilde{V}' e^{-i(1+C_V)\Gamma[\tilde{V}']}, \qquad (2.42)$$

$$Z_{\beta'} = \int \mathcal{D}\beta' e^{i\Gamma[\beta'] + i\lambda^2 \int \frac{1}{2} tr[\partial_-^{-1}(\beta'\partial_-\beta'^{-1})]^2} \qquad (2.43)$$

and

$$\tilde{V}' = V\beta'. \qquad (2.44)$$

Note that β' satisfies a different dynamics than β introduced previously.

It is convenient to rewrite Ω_- in eq. (2.7) in the form

$$\Omega_- = -\frac{1}{4e^2} V\partial_-(V^{-1}[\partial_+(Vi\partial_- V^{-1})]V)V^{-1} - (1+c_V)J_-(V) + j_-. \qquad (2.45)$$

Rewriting Ω_- in terms of E' by making use of the equation of motion

$$E' = -\frac{1}{2e}\partial_-(V^{-1}i\partial_+V) \tag{2.46}$$

and making use of (2.40), one arrives at

$$\Omega_- = -(1+c_V)J_-(\tilde{V}') + j_-. \tag{2.47}$$

We conclude that the corresponding BRST charge

$$Q_- = \int dx^1 trc_-^{(0)}[-(1+c_V)J_-(\tilde{V}') + j_- - \frac{1}{2}\{b_-^{(0)}, c_-^{(0)}\}] \tag{2.48}$$

must annihilate the physical states.

Notice that expression (2.47) formally resembles $\tilde{\Omega}_-$ of ref.[4]. Although $\tilde{V}$ in (2.38) and $\tilde{V}'$ in (2.48) obey the same dynamics, they are, however, vinculated by different constraints to the "massive" sector described in terms of the group-valued fields β and β', respectively, which in turn obey a different dynamics. The constraint $\hat{\Omega}_+ \approx 0$ associated with the change of variable $E' \to \beta'$ is again obtained following the previous systematic procedure, and one finds

$$\begin{aligned}\hat{\Omega}_+ &= -\lambda^2\beta'^{-1}(\partial_-^{-2}(\beta' i\partial_-\beta'^{-1}))\beta' + J_+(\beta') \\ &\quad -(1+c_V)J_+(\tilde{V}') + \{\hat{b}_+^{(0)}, \hat{c}_+^{(0)}\}.\end{aligned} \tag{2.49}$$

For similar reasons as before, the corresponding Noether charge

$$\hat{Q}_+ = \int dx' tr\hat{c}_+^{(0)}\left[\hat{\Omega}_+ - \frac{1}{2}\left\{\hat{b}_+^{(0)}, \hat{c}_+^{(0)}\right\}\right] \tag{2.50}$$

must annihilate the physical states.

On the ghost number zero sector, the BRST conditions to be imposed on the physical states are equivalent to requiring

$$\Omega_\pm \approx 0, \quad \hat{\Omega}_\pm \approx 0, \tag{2.51}$$

with $\hat{\Omega}_\pm$ and $\Omega_\pm$ given by eqs.(2.32), (2.37), (2.47) and (2.49).

3 The physical Hilbert space

In order to address the cohomology problem defining the physical Hilbert space, we must express the constraints in terms of canonically conjugate variables. To this end we first rewrite the partition function Z_β in (2.19) in terms of an auxiliary field B as follows:

$$Z_\beta = \int \mathcal{D}B\mathcal{D}\beta e^{iS[\beta,B]}, \tag{3.1}$$

where

$$S[\beta,B]=\Gamma[\beta]+\int tr\left[\frac{1}{2}(\partial_+B)^2+\lambda B\beta^{-1}i\partial_+\beta\right]. \tag{3.2}$$

Correspondingly we have for $Z_{\beta'}$ in (2.43)

$$Z_{\beta'}=\int \mathcal{D}B'\mathcal{D}\beta' e^{iS'[\beta',B']}, \tag{3.3}$$

with

$$S'[\beta',B']=\Gamma[\beta']+\int tr\left[\frac{1}{2}(\partial_-B')^2+\lambda B'\beta' i\partial_-\beta'^{-1}\right]. \tag{3.4}$$

We may then rewrite the constraints $\hat{\Omega}_\pm\approx 0$ in (2.32) and (2.49) as

$$\hat{\Omega}_-=\lambda\beta B\beta^{-1}+\frac{1}{4\pi}\beta i\partial_-\beta^{-1}-\frac{(1+c_V)}{4\pi}\tilde{V}i\partial_-\tilde{V}^{-1}+\{\hat{b}_-^{(0)},\hat{c}_-^{(0)}\}, \tag{3.5}$$

$$\hat{\Omega}_+=\lambda\beta'^{-1}B'\beta'+\frac{1}{4\pi}\beta'^{-1}i\partial_+\beta'-\frac{(1+c_V)}{4\pi}\tilde{V}'^{-1}i\partial_+\tilde{V}'+\{\hat{b}_+^{(0)},\hat{c}_+^{(0)}\}. \tag{3.6}$$

Define (tilde stands for "transpose")

$$\begin{aligned}
\tilde{\hat{\Pi}}^{(\beta)} &= \frac{1}{4\pi}\partial_0\beta^{-1}+i\lambda B\beta^{-1},\\
\tilde{\hat{\Pi}}^{(\beta')} &= \frac{1}{4\pi}\partial_0\beta'^{-1}-i\lambda\beta'^{-1}B',\\
\tilde{\hat{\Pi}}^{(\tilde{V})} &= -\frac{1+c_V}{4\pi}\partial_0\tilde{V}^{-1},\\
\tilde{\hat{\Pi}}^{(\tilde{V}')} &= -\frac{1+c_V}{4\pi}\partial_0\tilde{V}'^{-1}.
\end{aligned} \tag{3.7}$$

Canonical quantization then implies the Poisson algebra (see ref.[11, 12] for derivation; g stands for a generic WZW field of level n)

$$\begin{aligned}
&\{g_{ij}(x),\hat{\Pi}^{(g)}_{kl}(y)\}_P=\delta_{ik}\delta_{jl}\delta(x^1-y^1),\\
&\{\hat{\Pi}^{(g)}_{ij}(x),\ \hat{\Pi}^{(g)}_{kl}(y)\}_P=-\frac{n}{4\pi}\left(\partial_1 g^{-1}_{jk}g^{-1}_{li}-g^{-1}_{jk}\partial_1 g^{-1}_{li}\right)\delta(x^1-y^1).
\end{aligned} \tag{3.8}$$

In terms of canonical variables, we have for the constraints (2.37), (2.47),

$$\begin{aligned}
\Omega_+ &= -i\tilde{\hat{\Pi}}^{(\tilde{V})}\tilde{V}-\frac{(1+c_V)}{4\pi}\tilde{V}^{-1}i\partial_1\tilde{V}+j_+,\\
\Omega_- &= i\tilde{V}'\tilde{\hat{\Pi}}^{(\tilde{V}')}+\frac{(1+c_V)}{4\pi}\tilde{V}'i\partial_1\tilde{V}'^{-1}+j_-
\end{aligned} \tag{3.9}$$

and for the constraints (3.5), (3.6),

$$\begin{aligned}
\hat{\Omega}_- \;=\;& i\beta\tilde{\hat{\Pi}}^{(\beta)}+i\tilde{V}\tilde{\hat{\Pi}}^{(\tilde{V})}-\frac{1}{4\pi}\beta i\partial_1\beta^{-1}\\
&+\frac{1+c_V}{4\pi}\tilde{V}i\partial_1\tilde{V}^{-1}+\{\hat{b}_-^{(0)},\hat{c}_-^{(0)}\},
\end{aligned} \tag{3.10}$$

$$\hat{\Omega}_+ = -i\hat{\tilde{\Pi}}^{(\beta')}\beta' - i\hat{\tilde{\Pi}}^{(\tilde{V}')}\tilde{V}' + \frac{1}{4\pi}\beta'^{-1}i\partial_1\beta' - \frac{(1+c_V)}{4\pi}\tilde{V}'^{-1}i\partial_1\tilde{V}' + \{\hat{b}_+^{(0)}, \hat{c}_+^{(0)}\}. \quad (3.11)$$

With the aid of the Poisson brackets (3.8) it is straightforward to verify that $\hat{\Omega}_+^a = tr(\hat{\Omega}t^a)$ and $\hat{\Omega}_-^a = tr(\hat{\Omega}t^a)$ are first class:

$$\left\{\hat{\Omega}_\pm^a(x), \hat{\Omega}_\pm^b(y)\right\}_P = -f_{abc}\hat{\Omega}_\pm^c\delta(x^1-y^1). \quad (3.12)$$

Hence the corresponding BRST charges are nilpotent. Similar properties are readily established for the remaining operators $\Omega_\pm$. Furthermore,

$$\begin{aligned} &\{\Omega_+(x), \hat{\Omega}_-(y)\}_P = 0, \\ &\{\Omega_-(x), \hat{\Omega}_+(y)\}_P = 0. \end{aligned} \quad (3.13)$$

The physical Hilbert space of the non-local formulation of QCD_2 is now obtained by solving the cohomology problem associated with the BRST charges $Q_\pm, \hat{Q}_\pm$ in the ghost-number zero sector. The solution of this problem is suggested by identifying this space with the space of gauge-invariant observables of the original theory defined by (2.1). It is interesting to note that the constraints $\hat{\Omega}_\pm \approx 0$ are implemented by any functional of V (and the fermions), thus implying that $\tilde{V}, \beta(\tilde{V}', \beta')$ can only occur in the combinations $\beta^{-1}\tilde{V}(\tilde{V}'\beta'^{-1})$. Indeed, making use of the Poisson brackets (3.8), we have

$$\begin{aligned} &\left\{\hat{\Omega}_-^a(x),\ \beta^{-1}(y)\right\}_P = i(\beta^{-1}(x)t^a)\delta(x^1-y^1), \\ &\left\{\hat{\Omega}_-^a(x),\ \tilde{V}(y)\right\}_P = -i(t^a\tilde{V}(y))\delta(x^1-y^1), \\ &\left\{\hat{\Omega}_+^a(x),\ \tilde{V}'(y)\right\}_P = +i(\tilde{V}'(x)t^a)\delta(x^1-y^1), \\ &\left\{\hat{\Omega}_+^a(x),\ \beta'^{-1}(y)\right\}_P = -i(t^a\beta'^{-1}(y))\delta(x^1-y^1). \end{aligned} \quad (3.14)$$

As for the other two constraints, $\Omega_+ \approx 0$ and $\Omega_- \approx 0$ linking the bosonic to the free fermion sector, they tell us in particular, that local fermionic bilinears should be constructed in terms of free fermions and the bosonic fields as follows:

$$\left(\psi_1^{(0)\dagger}\beta^{-1}\tilde{V}\psi_2^{(0)}\right) = \left(\psi_1^{(0)\dagger}\tilde{V}'\beta'^{-1}\psi_2^{(0)}\right) = \left(\psi_1^{(0)\dagger}V\psi_2^{(0)}\right) = \left(\psi_1^\dagger\psi_2\right). \quad (3.15)$$

This is in agreement with our expectations.

4 The QCD_2 Vacuum revisited

The constraints, $\Omega_+ \approx 0$ and $\Omega_- \approx 0$, link the $\tilde{V}$-free fermions-ghosts and $\tilde{V}'$-free fermions-ghosts sectors respectively. They operate in the topological sector associated with the coset $U(N)_1/SU(N)_1$.

Before proceeding to the solution of the cohomology problem in this sector, one comment is in order concerning the factorization of the $U(1)$ degree of freedom. In fact, the factor $Z_{coset} = Z_F^{(0)} Z_{gh}^{(0)} Z_{\tilde{V}}$ in (2.17), corresponds to the partition function of the coset $U(N)/SU(N)_1 = U(1) \times SU(N)_1/SU(N)_1$ [13]. By bosonizing the free fermions [7] one can factorize the $U(1)$ degree of freedom, which shows that it merely acts as a spectator. (This factorization can no longer be done in the case of more than one flavor, leading to higher level $SU(N)$ affine Lie algebras).

The solution of the cohomology problem for the topological coset $SU(N)_1/SU(N)_1$ leads to the existence of N inequivalent vacua [14]. Each of these can be associated with a $SU(N)_1$ primary field. There are N such primary fields in the $SU(N)_1$ conformal quantum field theory, each one corresponding to a so-called integrable representation. The restriction in the number of the allowed representations arises from the affine (Kac-Moody) selection rules [15]. The construction of such primaries in the $SU(N)_1 = U(N)/U(1)$ fermionic coset theory has been carried out in ref.[13].

By further gauging the $SU(N)_1$ group we can show that these primaries are mapped into primaries of the coset $SU(N)_1/SU(N)_1$ of conformal dimension zero. These primaries, acting on the Fock vacuum, create the different inequivalent vacua of the topological coset theory. For the $U(N)/SU(N)_1$ coset the conformal dimension of the primaries is different from zero and is determined by the extra $U(1)$ factor. They are given in terms of the properly antisymmetrized product of p fermionic bilinears, $p = 1, ..., N$, which in terms of the decoupled fields read:

$$\Phi_p(z,\bar{z}) =: e^{2pi\phi} :: \psi_2^{(0)\dagger i_1} ... \psi_2^{(0)\dagger i_p} :: \psi_1^{(0)j_1} ... \psi_1^{(0)j_p} :: V_{\mathcal{A}}^{-1\ i_1 j_1 ... i_p j_p} :, \tag{4.1}$$

where

$$V_{\mathcal{A}}^{i_1 j_1 ... i_r j_r} \equiv [: v^{i_1 j_1} \dots v^{i_r j_r} :]_{\mathcal{A}} \tag{4.2}$$

Here v stands for $\tilde{V}$ or $\tilde{V}'$ (depending on the coset in question), and the subscript $\mathcal{A}$ means antisymmetrization in the left and right indices, separately. The conformal dimension of $V_{\mathcal{A}}$ is the conformal dimension of an $SU(N)_1$ primary field in the representation Λ_p whose Young diagram has p vertical boxes, as given by [16]

$$h_{\Lambda_p} = \bar{h}_{\Lambda_p} = \frac{c_{\Lambda_p}}{c_V + k} \tag{4.3}$$

where $c_V = N$ for $SU(N)$, $k = 1$ and $c_{\Lambda_p} = \frac{p}{2N}(N+1)(N-p)$, is the Casimir of the representation Λ_p. The additional vertex operator $: e^{2pi\phi} :$ is a result of the factorization of the $U(1)$ spectator as explained above. It should be stressed that this vertex operator (with conformal dimensions given by $h = \bar{h} = -p^2/2N$), is crucial to obtain the correct dimension of the primaries. They are the intertwining operators linking the N vacua of the conformal sector, referred to above.

In the non-conformal sector the primaries (4.1) are replaced by the properly antisymmetrized product of p fermionic bilinears,

$$\Phi_p(z,\bar{z}) = Tr\ \mathcal{A}\left(:\psi_2^\dagger\psi_1\psi_2^\dagger\psi_1 ... \psi_2^\dagger\psi_1:\right), \quad p = 1, ..., N, \tag{4.4}$$

which in terms of the decoupled fields are given by (4.1) with the replacement $v \to V$ in (4.2). The primaries (4.4) implement the constraints $\Omega_\pm \approx 0$, and thus create physical states.

If we assume the QCD_2 vacuum to lie in the conformal ($\beta = 1$) sector, then we must conclude that there exists an N-fold degeneracy of the QCD_2 ground state. This generalizes the conclusion of ref.[6], where this degeneracy has been discussed in some detail for the case of N = 2.

5 Conclusion

The main objective of this paper was to clarify the role of the various BRST symmetries and associated nilpotent charges present in the decoupled formulation of QCD_2. Our analysis has shown that of the three nilpotent charges obtained in ref.[4] in the non-local formulation, only two are required to vanish on $\mathcal{H}_{phys}$. They correspond to $\tilde{\Omega}_+ \approx 0$ and $\Omega \approx 0$ as given by equations (3.13) and (3.18) of that reference, or $\Omega_+ \approx 0$ and $\hat{\Omega}_- \approx 0$ in the notation of this paper. These constraints are first class. We have further shown that the constraint $\tilde{\Omega}_-$ in eq.(3.13) of [4] is to be replaced by the constraints $\Omega_- \approx 0, \hat{\Omega}_+ \approx 0$ in the present notation. These again represent a first class system. All these constraints were found to be implemented consistently by suitable products of fermion bilinears, corresponding to gauge invariant observables of the original partition function (2.1). This solves the corresponding cohomology problem in the ghost number zero sector.

The constraints $\hat{\Omega}_+ \approx 0$ and $\hat{\Omega}_- \approx 0$ couple the conformal sector of the theory to the sector of massive excitations β and β', whose dynamics is described by the partition function Z_β and $Z_{\beta'}$ in (2.19) and (2.43), respectively. Assuming that the QCD_2 ground state lies in the zero-mass, conformal sector, one is led to the conclusion that it is N-fold degenerate in the case of an SU(N) gauge symmetry with one flavor. This is in accordance with the conclusion reached in ref.[6], but is valid only, provided β and β' act as identity operators in this sector.

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